First characterizations of a Ring Cavity for a colt atom gravimeter

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1 Calculation of beamparamters

1.1 Gerneral parameters

The in the experiment used ring cavity consists out of three high reflective mirrors, like shown in figure 1.1.

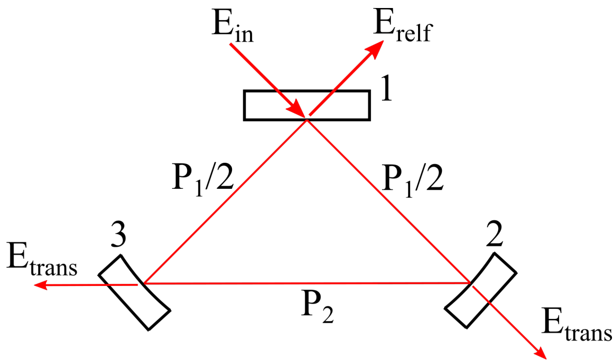

Figure 1.1: Schematic grafic of the ring cavity. The light gets transimitted into the ring cavity through mirror 1, indicated by E_{in}. The beams path's between mirror 1, 2 and 3 are named P_1 P_2. Throug all three mirrors light gets transmitted out of the cavity.

Mirror 1 is planar, mirror 2 and 3 have a radius of curvature of $r = 5\,\text{cm}$. Total light traveling length is $L = P_1 + P_2/2 + P_2/2 = 3.8\,\text{cm}$.

P_1 and P_2 form a right triangle, with it's corner on mirror 1. Therefore is $P_1/2 = c = L/(\sqrt{2}+2)$ and $P_2 = L/(\sqrt{2}+1)$.

It's free spectral range is $\nu_0 = c/L = 7.889\,\text{GHz}$, where c is the speed of light. The mirrors do at the moment of the internship (January 2015) have reflectivities of $R_1 = 0.997$ and $R_{2,3} = 0.999$. After one circulation in the cavity the electric lightfield is becuse of tranmission losses damped by the factor $r_d = \sqrt{R_1 R_2^2}$. The finesse F gives a value for the losses of a resonator, which can be calculated out of the damping factor r_d over

$$F = \pi \frac{\sqrt{r_d}}{1 - r_d}. \tag{1.1}$$

For the given reflectiveties the finesse is $F = 1256$[1]. The higher the finesse, the slower the lightfield in the cavity decayes. In a more developed state of the experiment the mirrors

[1]Source: 'Angewandte Optik und Laserphysik', lecture of Prof. Claus Zimmerman, University Tübingen, 2013

will be changed to one's with higher reflectivities and a finesse of $F \approx 150000$ will be needed to reach.
Figure 1.2 shows the resonators groundmode with modulated sidebands to measure it's finesse.

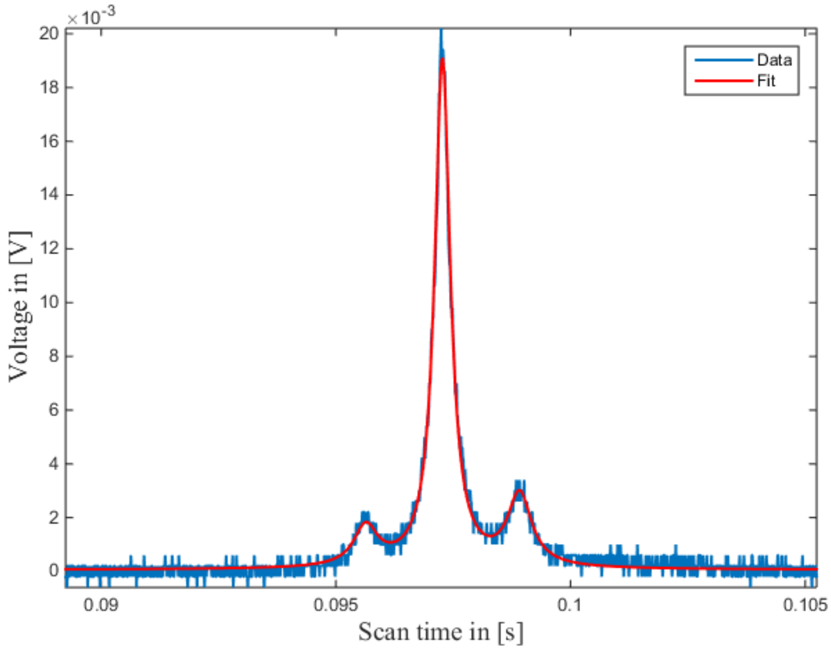

Figure 1.2: Modulated side band's to measure linewith and finesse. The diode lasers laser diode current was modulated with $f_{\mathrm{mod}} = 20\,\mathrm{MHz}$. The resonators full linewidth is therefore $\nu_{\mathrm{FWHM}} = 4.92\,\mathrm{MHz}$ which corresponds to a finesse of $F \approx 1600$.

1.2 Gaussian parameters of the ring cavity

A gaussian beam can be characterised with the confocal parameter z_0. It's value gives the distance from the focal point of a gaussian beam to the point where the radius of cuvature of the beams wavefronts is at it's minimum, or the radius ω of the beam is $\omega = \sqrt{2}\omega_0$, where ω_0 is the beams waist diameter.
z gives the position in a beam, relativ to the waist ω_0. z and z_0 are combined to the value $q = z + iz_0$.[2]
The ray's path throuh a optical system can be described with transfer matrices

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{1.2}$$

Depending on the system the ray's parameters are changed to

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}, \tag{1.3}$$

with the beams parameters q_1 infront of a the optical system and q_2 after it.

[2]Soruce: H. Kogelnik and T. Li, "Laser Beams and Resonators," Appl. Opt. 5, 1550-1567 (1966)

Reflektion on a mirror with radius of curcvature R ist given by

$$M_R = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \tag{1.4}$$

and translation through space by distance d with

$$M_d = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}. \tag{1.5}$$

If the ray circulates in the ring cavity, after getting reflected on the mirrors and traveling the space between them, it needs to be projected on itself, expressed by $q_2 = q_1 = q$. Equation 1.3 can then be solved for q, which yield's

$$q = \frac{A-D}{2C} \pm \frac{i}{2C}\sqrt{4-(A+D)^2}, \tag{1.6}$$

by using the relation for the determinant $AD - CB = 1$. With equation 1.6 z_0 can be identified as

$$z_0 = \frac{1}{2C}\sqrt{4-(A+D)^2}. \tag{1.7}$$

z_0 is now the confocal parameter of the resonator, depending on the path between the mirror's which is been looked at. Furthermore can the waist of each path be calculated by

$$\omega_0 = \sqrt{\frac{\lambda z_0}{\pi}}, \tag{1.8}$$

with the wavelength λ of the incoming light.

We now parametrize the beam into the rightangular coordinates x- and y, wehre the resonators plane (the triangle described by the resonaotrs mirrors, the horrizontal plane if the resonaots triangle is parralel to optical table) is the x-coordinate and y the one perpendicular to x.
The effective radius of curvature seen by a beam getting reflected on a spherical mirror depends on it's angel of incident onto the mirrors surface. If the angle of incidents on the mirrors plane is α, then the effective radii are

$$R_x = \frac{R}{\cos(\alpha)}, \tag{1.9}$$

and

$$R_y = R\cos(\alpha). \tag{1.10}$$

For both mirror 2 and 3 the angle of incidence is $\alpha = 22.5°$. Mirror 1 is planar, which equals to an infinite radius of curfature and the matrix that represents reflection on it becomes a identity matrix.
The resonators modes are because of this reason eliptical. x- and y-beam have to be watched at independently.
Now it is possible to calculate all the resonaotrs parameters. Because of the symetrical mirrors, the resonator has two beamwaists. One laying on mirror 1 and the second one exactly between mirror 2 and 3.

	$\omega_0/\mu m$		z_0/cm	
Path	**x**	**y**	**x**	**y**
P_2	70.5	68.09	2.27	2.11
P_1	64.24	60.36	1.88	1.66

Table 1.1: Table show's beamwaist ω_0 and confocal parameter z_0 for path P_2 between mirror 2 and 3 and for path P_1 between mirror 3 and 1 and 1 and 2.

1.3 Modespectrum

The bevore calculated parameters are valid for the groundmode of the resonator. The lights electric field has an infinite amount of independend pathes it can circulate on.
The solution for the differential equation that describes the allowed field distributions, is given by the hermite polynomials. The modes are characterises by their order. m is identivied as the modes order in x-direction, n as the modes order in y-direction. So each mode is identified with a pair of nautral numbers (m,n).

1.3.1 Gouy-Phase

Each mode collects a slitely different phase when doing one circulation. There is the geometrical phase kz, with the wavevektor k and the traveled distance z. And there is the Gouy-Phase.[3] The geometrical difference of a gaussian beam compared to an plain wave leads to phase difference of a maximum of π. The Gouy-Phase of every mode is calculated as

$$\theta = \frac{1}{2}\left((1+2m)\arctan\left(\frac{z}{z_{0,x}}\right) + (1+2n)\arctan\left(\frac{z}{z_{0,y}}\right)\right) \tag{1.11}$$

The toatal Gouy-Phase collected in the resonator is the summ of the Gouy-Phase of each path.

$$\begin{aligned}\theta &= \theta_{P_1} + \theta_{P_2} \\ &= \left((1+2m)\arctan\left(\frac{L/(\sqrt{2}+2)}{z_{0,P_1,x}}\right) + (1+2n)\arctan\left(\frac{L/(\sqrt{2}+2)}{z_{0,P_1,y}}\right)\right) \\ &+ \left((1+2m)\arctan\left(\frac{L/2(\sqrt{2}+1)}{z_{0,P_2,x}}\right) + (1+2n)\arctan\left(\frac{L/2(\sqrt{2}+1)}{z_{0,P_2,y}}\right)\right)\end{aligned} \tag{1.12}$$

1.3.2 Mirror's π-shift

The lights electric field amplitude changes it's algebraic sign when refleced on an optical dense medium. If it gets reflected on an uneven number of mirrors, the resulting phaseshift

[3] Source: 'Angewandte Optik und Laserphysik', lecture of Prof. Claus Zimmerman, University Tübingen, 2013

is π. Only horizontal modes of uneven order are affected[4]. This geometric phaseshift can be expressed by

$$\varphi_{\mathrm{geom}} = \frac{\pi}{2}(1-(-1)^m). \tag{1.13}$$

1.3.3 Resonace frequencies

Total phase after one circulation is then

$$\varphi = -kL + \theta(z) + \varphi_{\mathrm{geom}} = 2\pi q, \tag{1.14}$$

with the total length L of the resonator. Because we are looking for the case of constructive interference, the total phase needs to be equal to $2\pi q$, with $q \in \mathbb{Z}$. Because of $k = 2\pi/\lambda$ and $c = \nu\lambda$, equation 1.14 can be solved for the resonance frequency ν of the resonaotors modes

$$\nu_{\mathrm{q,m,n}} = \frac{c}{2\pi L}(\theta + \varphi_{\mathrm{geom}} - 2\pi q). \tag{1.15}$$

[4] Hermite polynomes of uneven order are antisymmetric.

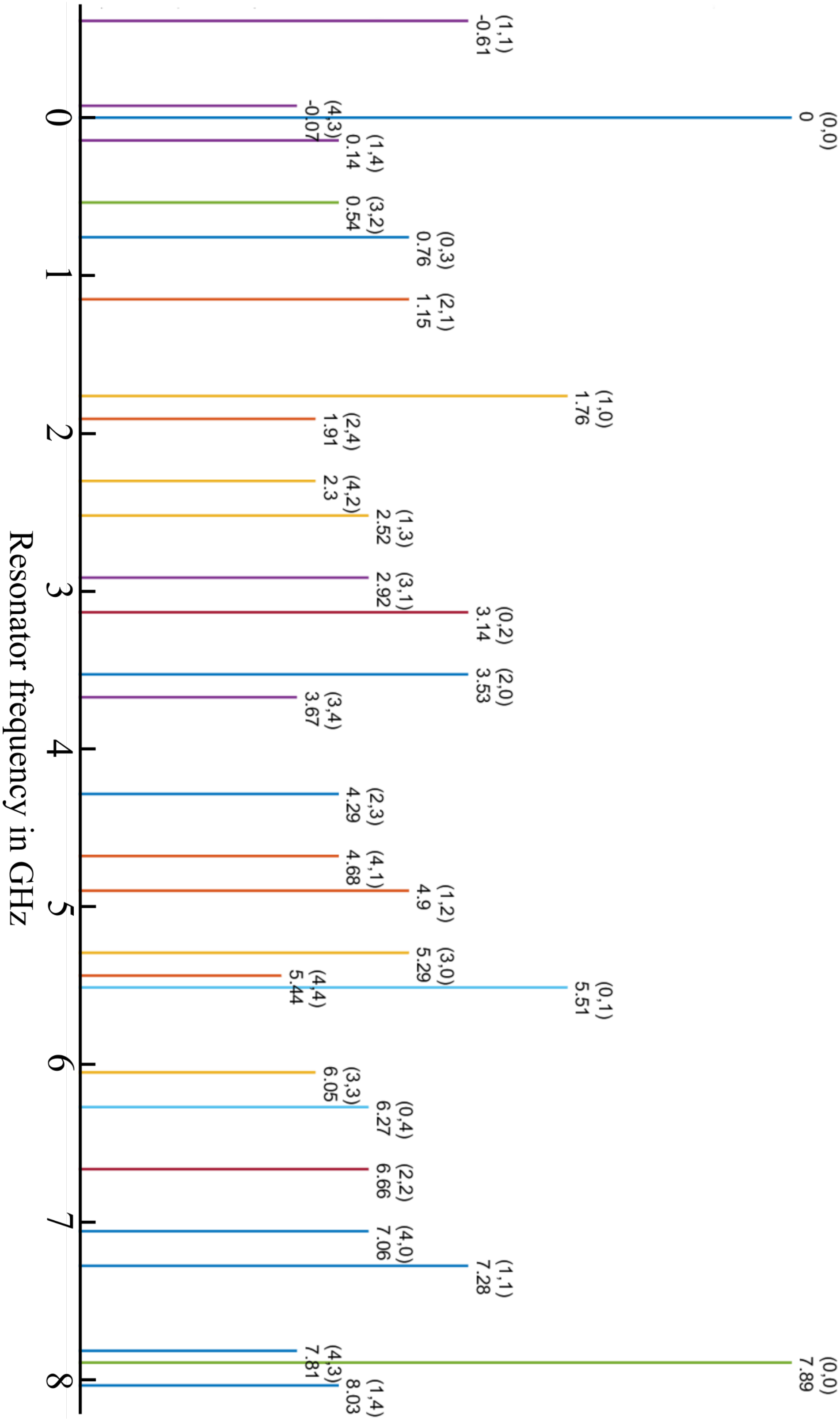

Figure 1.3: Calculated spectrum of resonators modes. The pair of numbers in the bracket show the modes orders. The number under the bracket show the like in eqaution 1.15 calculated modes frequency.

1.3.4 s- and p-polarisation

The plane of incident onto the mirrors surface is the resonators triangle, or in this case the laboratories optic table. Linear polarised light perpendicular to the plane of incidence is called s-polarised[5]. It's electric field amplitude vector is parallel to the mirrors plane. p-polarised lights electric field amplitude is parallel to the plane of incidence.
The mirrors dielectric layers are designed, so that p-polarised light has a much higher reflectivity than s-polarised. Therefor by p-polarised light exited resonator modes are called high finesse modes. The ones exited by s-polarised light are called low-finesse modes.
Because the electric field of p-polarised light is changing it's algebraic sign at every reflection, it's total phase difference after one circulation relative to s-polarised light, of which the field's amplitude is not changing it's algebraic sign at reflection, is then π.
Meaning, that high-finesse modes are shifted by half a free spectral range relative to the low finesse modes.

[5]'s' from the german 'senkrecht' (perpendicular)

2 Measuring the mode spectrum

In the experiment a diode laser with $\lambda = 689nm$ is used.[1] Used optical layout is shown in figure 2.1. The setup was also used to optain a Pound-Drever-Hall lock signal.
The diode laser showed the tendency to lose it's lasing threshold after a couple of days, and had to be readjusted. This was noticable by a increasing frequency stabilety. The reflected intensity dips of the resonator's modes showed more and more noise. Also the current interval of the lasers diode current, in which it was operating in single mode decreasing fast. The thresholds readjustment could be done quite fast using the fine-thread screw of the diode lasers optical grid.

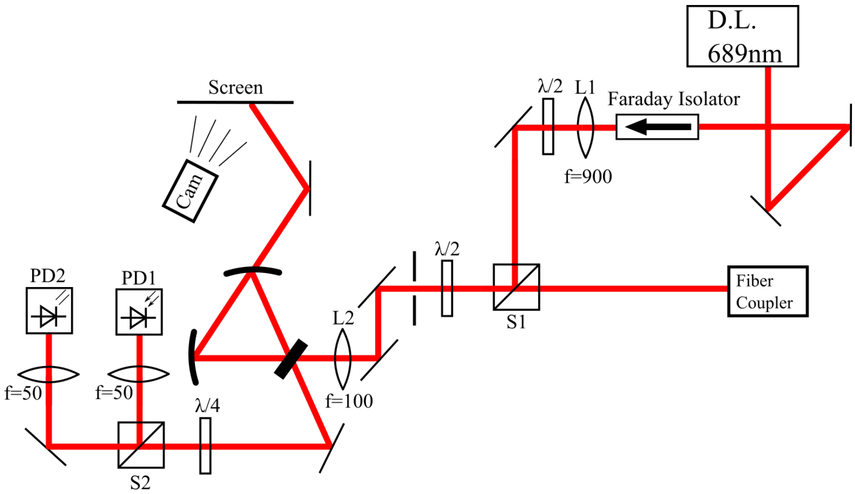

Figure 2.1: Optical scheme that was used. The two light sources could be used independently, a selfe made diode laser ore over the fiber an Toptica DL Pro. To prefent reflections back into the laser diode, a faraday isolator was put directly after the diode laser's output. Lens $L1$ is the first beam correction lense. Polcube $S1$ was used bring both lightsources into the same path. With $\lambda/2$-Plate after $S1$ s- and p-polarisation can be selected. A webcam was used to make pictures of the resonators modes, which have been projected onto an screen. Photodiode's $PD1$ and $PD2$ are used the monitor the reflected beam and to optain a Pound-Drever-Hall lock signal. $L2$ is the second mode matching lens, to fit the resonators modes.

Figure 2.3-2.5 show several horizontal and vertical modes with their measured frequency. The normalised reflected intensity is plotted over the resonators frequency over more then one free spectral range. The shown frequencies values are not verry accurate and should only be considered as a first look. The intensity was measured with an oszilloskope in timedomain. The horizontal time-axis was simply rescaled, takeing the distance between

[1]"Creation of a narrow linewidth, high passive stability laser for use in an ultra-cold strontium experiment", R. del Aguila, Internship report

the two $(0,0)$-groundmodes as reference for one free spectral range $\nu_0 = 7.889\,\mathrm{GHz}$. Non-linear piezo elongation and voltage have not been taken account of.

A mode matching efficiency of up to 80% could be reached, but was mostly around 70%, see figure 2.2.

All the shown pictures show the high-finesse modes (p-polarisation). The low finesse spectrum is expected to look the same and beeing shifted by a half free spectral length relative to the high finesse modes.

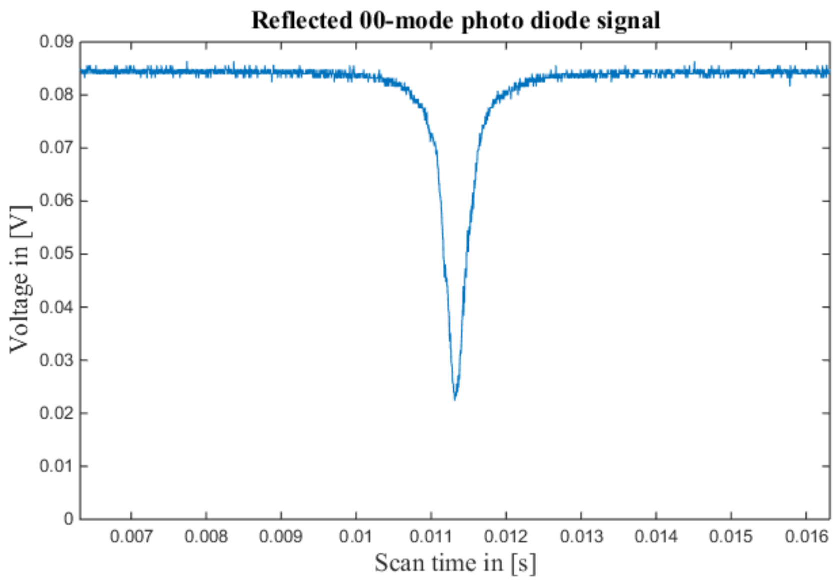

Figure 2.2: Of ring resonator reflected intensity, while scaning over the resonaotors mode with a piezo.

If the measured mode frequencies in figure 2.3-2.5 and the calculated in figure 1.3 are compared, one can see that most values fit quite well. Differences from up to 0.5 GHz apear, which is around 6.4% of one free specrtal range.

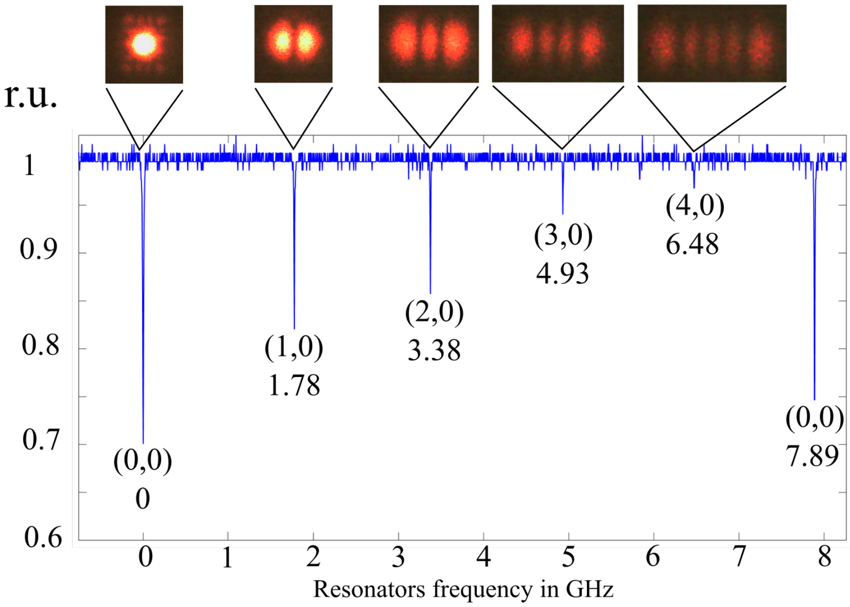

Figure 2.3: Horizontal axis shows the resonators frequency. Vertical axis is the normalised reflected resonaotrs intensity. Only horizontal modes are excited. The numbers in the bracket show the modes order (m, n). The number under the bracket is the measured frequency of the mode in GHz.

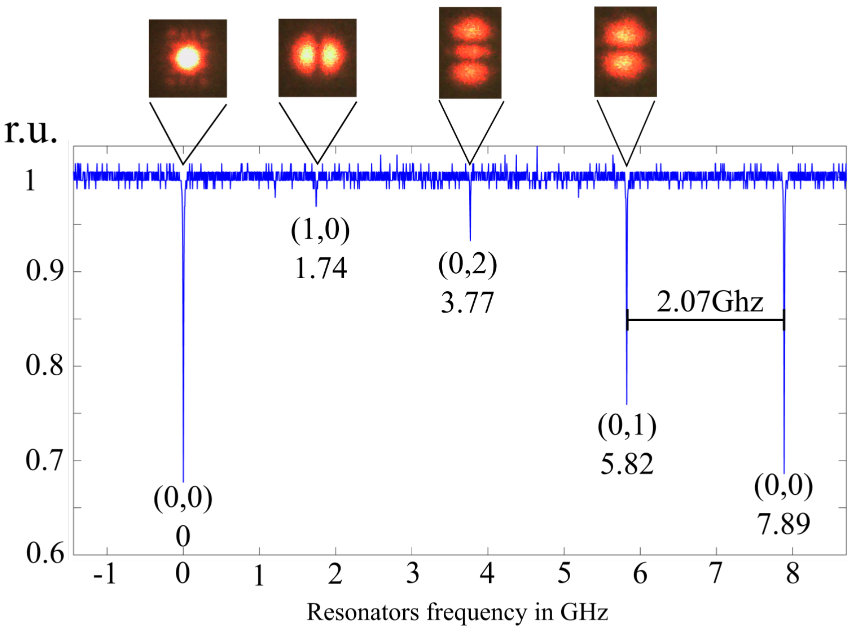

Figure 2.4: Horizontal axis shows the resonators frequency. Vertical axis is the normalised reflected resonaotrs intensity. Mostly vertical modes are excited.

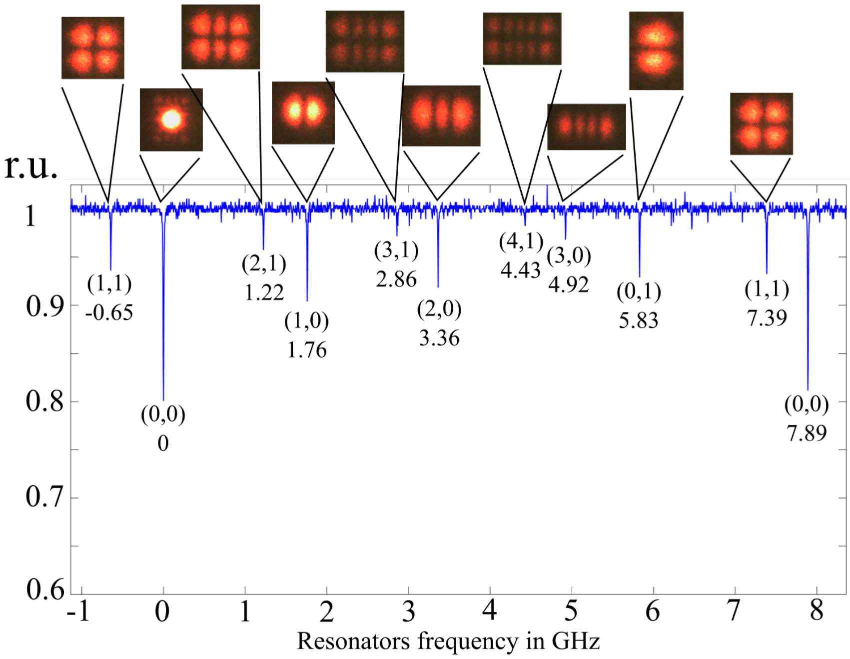

Figure 2.5: Horizontal axis shows the resonators frequency. Vertical axis is the normalised reflected resonaotrs intensity. Mostly higher modes of poth directions are excited.

2.1 Pound-Drever-Hall signal

A pound drever hall lock signal could be generated.[2][3] The laser diodes current was modulated with $V_{\text{mod}} = 310\,\text{mV}$ and a modulation frequency of $f_{\text{mod}} = 25\,\text{MHz}$. The photo diode's (figure 2.1) signal was mixed with the modulation frequency with an amplitude $V_{mix} = 410\,\text{mV}$. The laser diodes current was at $I_{LD} = 60.1\,\text{mA}$. The phase difference between modulation and reference signal has been $\varphi = 140°$. Figure 2.6 shows the generated PDH-signal. The reason for it's asymetric shape could be a second higher mode that lays at near to the groundmode. The picture of the groundmode for example in figure 2.3 shows the 34-mode together with the groundmode.

[2]Source: "Pound-Drever-Hall locking of an optical cavity for use in an Ultra-Cold Strontium Experiment", Faizah Rafique, Internship report

[3]Source: "An introduction to Pound–Drever–Hall laser frequency stabilization", Eric D. Black, Am. J. Phys. 69, 79 (2001)

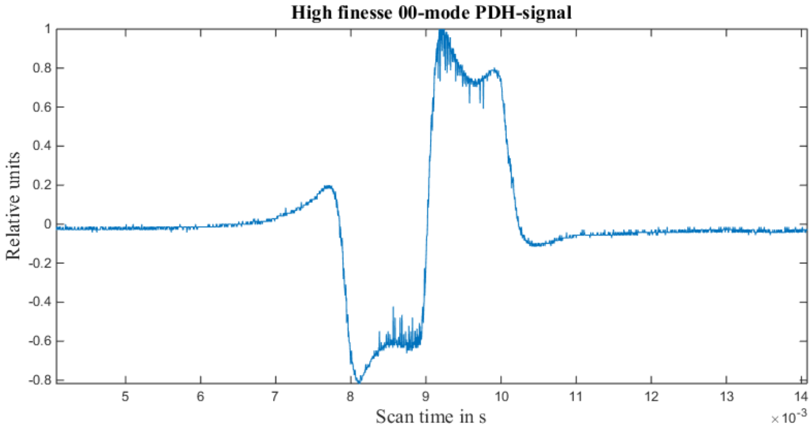

Figure 2.6: Measured Poun-Drever-Hall signal of the high finesse 00-Groundmode. The asymetries offspring is possebly the low finesse 34-Mode laying next to high finesse groundmode.